A MYSTIFYING THING

QUIXOTIC EVIL

Definition 1.

$$! : \mathbf{N} \mapsto \mathbf{N}$$
$$n! = \begin{cases} 1 & (n = 0) \\ n * (n-1)! & (n > 0) \end{cases}$$

Theorem 1.

$$n! \geq n^2, \forall n \in \mathbf{N}$$

Proof.

$$0! = 1, 0^2 = 0$$

Hence:

$$n! \geq n^2, n = 0 \tag{1}$$

Similarly:

$$1! = 1, 1^2 = 1$$
$$n! \geq n^2, n = 1 \tag{2}$$

Let $\exists m \in \mathbf{N}, m > 1$:

$$m! \geq m^2 \tag{3}$$

Assume:

$$(m+1)! < (m+1)^2$$

Then:

$$m!(m+1) < (m+1)(m+1)$$
$$m! < m+1$$

By (3):

$$m^2 < m+1$$

But $m > 1$, so $m^2 > m + 1$.
Therefore with (3):

$$n! \geq n^2 \Rightarrow (n+1)! \geq (n+1)^2, \forall n > 1, n \in \mathbf{N} \tag{4}$$

Result is by induction on (1),(2),(4). □

Date: January 11, 2007.